$$B(\varepsilon_0) = \{(x_1, x_3) : \max(|x_1 - d_1|; |x_3 - d_3|) \leqslant \varepsilon_0; \quad (d_1, d_3) \in \Gamma(\mathrm{Pr}_{x_1ox_3}[E])\}$$

$$K(\varepsilon_0) = B(\varepsilon_0) \cap \mathrm{Pr}_{x_1ox_3}[E], \quad U(\varepsilon_0) = \mathrm{Pr}_{x_1ox_3}[E] \setminus K(\varepsilon_0)$$

Taking the definition of the norm $\|x\| = \max_{1, \ldots, 4} |x_i|$ into account, we find from the previous constructions, that we can take as G any open bounded connected set in R_x^4 such that

$$\mathrm{Pr}_{x_1ox_3}[G] = U(\varepsilon_0)$$

Naturally G is nonempty because ε_0 satisfies the conditions (3.4). Thus all conditions of Theorem 1 hold. Consequently, no matter how small the positive number ε_0 and what continuously differentiable functions $u_1(t)$ and $u_2(t)$ are chosen, no Liapunov-stable motion of (3.1) exists belonging, at all $t \geqslant 0$, to the set G constructed.

The author thanks V.V.Rumiantsev for valuable advice.

REFERENCES

1. Chetaev, N.G., On the stable dynamic trajectories. Sb.nauchn.tr.Kasansk. aviats.inst., № 5, 1936.
2. Chetaev, N.G., On the stable dynamic trajectories. Uch.zap.Kazansk.un-ta, Vol.91, Book 4, № 1, 1931.
3. Chetaev, N.G., Stability of Motion. (English transaltion), Pergamon Press, Book № 09505, 1961.
4. Kostiukovskii, Iu.M.-L., On an idea of Chetaev. PMM Vol.37, № 1, 1973.
5. Bendixon, I., Sur les courbes definies par des équations différentielles. Acta Math., Bd. 24, S. 1–88, 1901.
6. Dulac, H., Recherche des cycles limites. Compt.rend., Acad.Sci., t.204, №23, p.1703–1706, 1937.
7. Blaquire, A., Nonlinear system analysis. New York - London, Acad.Press, 1966.

Translated by L.K.

UDC 533.6.011.8

ON THE PROBLEM OF STUDYING EXACT SOLUTIONS OF A SYSTEM OF EQUATIONS OF KINETIC MOMENTS

PMM Vol.38, № 2, 1974, pp.369-372
V.Ia.RUDIAK
(Novosibirsk)
(Received 21 November 1972)

Galkin in his papers [1, 2] obtained a class of exact solutions of the system of kinetic moments for a monatomic Maxwell-type gas. The simplest flows described by these solutions, namely the shear and divergent flows, were used to analyze the domain of applicability of the Chapman-Enskog method [1, 3, 4]. The present paper deals with certain other flows belonging to this class. The solutions obtained are used to investigate the domain of applicability of the Navier-Stokes and Barnett approximations to the Chapman-Enskog method.

1. Let us consider a one-dimensional flow for which the macroscopic velocity components u_x, u_y and u_z depend on the coordinates x, y, z and time t in the following manner (c is a constant):

$$u_x = x/(t+c) + y/(t+c), \qquad u_y = u_z = 0 \tag{1.1}$$

Solving the equation of continuity for this flow leads to the following expression for the density ρ:

$$\rho = \rho(0)/\tau, \qquad \tau = 1 + t/c \tag{1.2}$$

where $\rho(0)$ is the initial value of the density.

Taking (1.2) into account, we can write the energy equation and the stresses p_{ij} ($i, j = x, y, z$) in the form

$$\begin{aligned}
&\frac{dp}{dt} + \frac{5}{3}\frac{p}{(t+c)} + \frac{2}{3}\frac{p_{xx}}{(t+c)} + \frac{2}{3}\frac{p_{xy}}{(t+c)} = 0 \\
&\frac{dp_{xx}}{dt} + \left[\frac{7}{3} + \alpha_0 c\right]\frac{p_{xx}}{(t+c)} + \frac{4}{3}\frac{p_{xy}}{(t+c)} + \frac{4}{3}\frac{p}{(t+c)} = 0 \\
&\frac{dp_{yy}}{dt} + [1 + \alpha_0 c]\frac{p_{yy}}{(t+c)} - \frac{2}{3}\frac{p_{xx}}{(t+c)} - \frac{2}{3}\frac{p_{xy}}{(t+c)} - \frac{2}{3}\frac{p}{(t+c)} = 0 \\
&\frac{dp_{xy}}{dt} + [2 + \alpha_0 c]\frac{p_{xy}}{(t+c)} + \frac{p_{yy}}{(t+c)} + \frac{p}{(t+c)} = 0 \\
&(\alpha_0 = R\rho(0)/\mu_0,\ R = p/\rho T,\ \mu_0 = \mu/T)
\end{aligned} \tag{1.3}$$

where p is pressure, T is temperature and μ is the coeffcient of viscosity. Solving the system (1.3) we can find the values of the components of the stress and pressure tensors. In particular, for p and p_{xx} we obtain

$$\frac{p}{p(0)} = A\tau^{\lambda/3} + \{B_+q_+ + B_-q_-\} \tag{1.4}$$

$$\frac{p_{xx}}{p(0)} = \frac{2(\lambda+3)}{\lambda+3+3/F}A\tau^{\lambda/3} + \{D\tau^{-1-1/F} + C_+q_+ + C_-q_-\} \tag{1.5}$$

$$\begin{aligned}
&q_\pm = \tau^{-\lambda/6-3-1/F}\exp\left[\pm\frac{i}{2\sqrt{3}}(u-v)\ln\tau\right] \\
&\lambda = u + v - 6 - 2/F \\
&u, v = [F^{-3} + {}^{3}/_{2}F^{-2} + {}^{27}/_{2}F^{-1} \pm ({}^{69}/_{4}F^{-4} + {}^{43}/_{2}F^{-3} + {}^{585}/_{4}F^{-2} - 27F^{-1} - 27)^{1/2}]^{1/3}
\end{aligned}$$

Here A, $B_\pm$ and $C_\pm$ are constants depending on the initial values of the problem, $p(0)$ is the initial pressure, $F = 1/(c\gamma_0) \sim \mathrm{Kn}$ and Kn is the Knudsen number.

Using the Chapman-Enskog method to solve the problem in question, we obtain the following results:

$$p/p_0 = \tau^{-5/3}, \qquad p_{xx} = 0 \tag{1.6}$$

in the Euler approximation,

$$p/p_0 = \tau^{-5/3 + 14/9 F}, \quad p_{xx}/p_0 = -{}^{4}/_{3}F\tau^{-5/3+14/9F} \tag{1.7}$$

in the Navier-Stokes approximation and

$$\begin{aligned}
&p/p_0 = \tau^{-5/3 + 14/9F - 34/27F^2} \\
&p_{xx}/p_0 = (-{}^{4}/_{3}F + {}^{20}/_{9}F^2)\tau^{-5/3+14/9F-34/27F^2}
\end{aligned} \tag{1.8}$$

in the Barnett approximation. In the above expressions p_0 represents a certain value of the initial pressure which must be specified when the Chapman-Enskog method is used. To make the Chapman-Enskog solution closed, it will be necessary to establish the relation connecting p_0 and the true initial pressure $p\ (0)$.

Comparing the expressions (1.6) – (1.8) with the exact solutions, we notice the absence in the Chapman-Ensskog solution of the terms which appear in the exact solutions within the braces. These terms however decrease rapidly if (τ_0 is the time of the mean free path of a molecule)

$$t \gg 1 / \alpha_0 = \tau_0, \qquad F < 1 \tag{1.9}$$

In this case the exact solutions assume the form

$$p/p_0 = A\tau^l, \quad p_{xx}/p\ (0) = A\left[-{}^4/_3 F + {}^{20}/_9 F^2 + O\ (F^3)\right] \tau^l \tag{1.10}$$
$$l = -{}^5/_3 + {}^{14}/_9 F - {}^{34}/_{27} F^2 + O\ (F^3)$$
$$A = 1 - {}^2/_3 F\ (\Pi_{xy} + \Pi_{xx}) + O\ (F^2), \qquad \Pi_{ij} = p_{ij}\ (0) / p\ (0)$$

Thus, according to the condition (1.9), the Chapman-Enskog method can be used outside the initial layer the dimension of which is of the order of the mean free path of the molecule, and at small values of the Knudsen number.

It is significant that the structure of the exact solution is different from the structure of the solution obtained by the Chapman-Enskog method. The exact solution depends, at any instant of time, on the initial parameters of the problem, through the constant A. It is only when $A = 1$ that the exact solution becomes a normal solution in the sense of Chapman-Enskog. Figure 1 shows the comparison of the solution (1.10) with the Chapman-Enskog solutions (1.7) and (1.8) for this particular case, and the dependence of $\xi = \ln\ (p\ /\ p\ (0))\ /\ \ln \tau$ on the parameter F is shown for a fixed instant of time and denoted by *1* for the exact solution, by *2* for the Navier-Stokes approximation and by *3* for the Barnett approximation. The computations performed show that the Barnett approximation, unlike the Navier-Stokes approximation, is in good agreement with the exact solution up to the values of the Knudsen number of the order of unity.

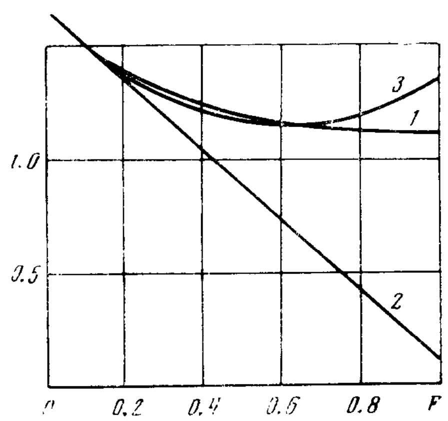

Fig. 1

Earlier we said that the Chapman-Enskog method of solution requires that the relation between $p\ (0)$ and p_0 be established. We can do this in the present case. In fact, when $A = 1$, the initial layer is absent and $p_0 = p\ (0)$. On the other hand, in the general case and for small F we can write the solution for p in the initial layer in the form

$$p\ /\ p\ (0) = A\tau^l + P\ (p\ (0), \Pi_{ij}, t, F)\ \tau^{-l,2-3-1,F} \tag{1.11}$$

Here p denotes the rapidly decaying part of the solution (1.4). The second part of the solution (1.11) decays rapidly, therefore we can write, in the sense of the condition (1.9), the following asymptotic expression: $p \to p\ (0)\ A\tau^l$. Consequently the initial value of p in the Chapman-Enskog method should be given such, that the relation

$$p_0 = p\,(0)\,[1 - {}^2/_3\,(\Pi_{xy} + \Pi_{xx})\,F + O\,(F^2)]$$

holds. A similar solution was obtained by Galkin [5] for one-dimensional divergence flow by matching internal and external expansions of the exact solution with respect to a small parameter F.

When the initial values are arbitrary and F is small but finite, the solutions obtained by using the Chapman-Enskog method are applicable over finite time intervals just as in the case of a shear flow [4] and a one-dimensional divergent flow [3], for the reason that at $t \to \infty$, the approximate values of p and p_{ij} may differ from the exact values by arbitrarily large amounts. It should however be noted that the Barnett approximation can be used over the time intervals of much greater length than the Navier-Stokes approximation. When $F = 0$, the Euler approximation of the Chapman-Enskog method is identical with the exact solution. In this case the initial layer is absent and the Chapman-Enskog solution is uniformly valid over the whole time interval extending from zero to infinity.

2. Let us now consider a plane divergent flow

$$u_x = x\,/\,(t + c),\quad u_y = y\,/\,(t + c),\quad u_z = 0,\quad \rho = \rho\,(0)\,/\,\tau^2$$

The system of equations of kinetic moments for a monatomic Maxwell-type gas reduces in the present case to a degenerate hypergeometric equation, and the solutions for p and p_{ij} can be written in terms of the degenerate hypergeometric functions [6, 7].

Solution of the problem in the Navier-Stokes and Barnett approximations to the Chapman-Enskog method, gives the following corresponding results:

$$p/p_0 = \tau^{-10/3} e^{8/9 F\tau} \tag{2.1}$$

$$p/p_0 = \tau^{-10/3} e^{8/9 F\tau - 8/27 F^2\tau^2} \tag{2.2}$$

The asymptotic behavior of the exact solution as $t \to \infty$ was studied in [6] where an asymptotic approach was used to show that the exact solution is asymptotically stable. At the same time the Navier-Stokes approximation (2.1) is found to be asymptotically unstable, i.e. although the density in the plane divergent flow decreases with time and the external forces are absent, the pressure and stresses increase. This implies that the Navier-Stokes approximation cannot be used to describe the flow in question, no matter what values the parameter F assumes. On the other hand, the Barnett approximation (2.2) yields an asymptotically stable solution. Thus the Barnett approximation is superior to the Navier-Stokes approximation and gives, generally speaking, a qualitatively correct result.

The same characteristic feature is associated with a flow obtained by superimposing the plane divergent and shear flows

$$u_x = x\,/\,\tau,\qquad u_y = xc\,/\,\tau^2 + y\,/\,\tau,\qquad u_z = 0$$

In fact, it can be shown that the exact solution describing this flow is asymptotically stable. Using now the Chapman-Enskog method to solve the problem in the Navier-Stokes approximation we obtain

$$p/p_0 = \tau^{-10/3} \exp\,({}^8/_9 F\tau - {}^2/_3 F\tau^{-1})$$

and $p \to \infty$ as $t \to \infty$.

The Barnett approximation, on the other hand, yields

$$p/p_0 = \tau^{-10/3 - 4/3 F^2} \exp\left({}^8/_9 F\tau - {}^2/_3 F\tau^{-1} - {}^4/_{27} F^2\tau^2 \right)$$

and the solution is asymptotically stable for any fixed value of F.

The author expresses deep gratitude to his supervisor V.V.Struminskii for formulating the problem and for valuable comments.

REFERENCES

1. Galkin, V.S., On a solution of the Boltzmann kinetic equation. PMM Vol.20, №3, 1956.
2. Galkin, V.S., On a class of solutions of Grad's kinetic moment equations. PMM Vol.22, №3, 1958.
3. Galkin, V.S., One-dimensional unsteady solution of the equations for the kinetic moments of a monatomic gas. PMM Vol.28, №1, 1964.
4. Truesdell, C., On the pressures and the flux of energy in a gas according to Maxwell's kinetic theory. J. Rational Mech. and Analysis, Vol.5, №1, 1956.
5. Galkin, V.S., On homogeneous flows of a rarefied gas at small Knudsen numbers. Izv. Akad. Nauk SSSR, MZhG, №2, 1969.
6. Borisov, A.S., Asymptotic behavior of certain solutions of systems of Grad kinetic moments. Inzh, zh., Vol.5, №6, 1965.
7. Galkin, V.S., On exact solutions of equations of kinetic moments for a mixture of monatomic gases. Izv. Akad. Nauk SSSR, MZhG, №5, 1966.

Translated by L.K.

UDC 539.3

SOME CONTACT PROBLEMS FOR A THREE-DIMENSIONAL WEDGE WITH A FINITE NUMBER OF CONTACT REGIONS

PMM Vol.38, №2, 1974, pp.373-377
V.N.BERKOVICH
(Rostov-on-Don)
(Received October 17, 1972)

Mixed problems for a three-dimensional wedge whose edge is unbounded on both sides are considered. The case of several contact sections between the wedge and the stamps is investigated. Theorems for solvability of the integral equations are established in a number of cases and the properties of their solutions are studied. Approximate formulas are obtained for small wedge angles.

The problem was examined in [1] in the case of one contact section, where the method elucidated in [2] was applied. The convolution integral equation given on a system of segments was studied in [3].

The equation of [1] on a system of segments is considered below according to the scheme elucidated in [3].

1. On the basis of the Ufliand solution [4] the antisymmetric mixed problem (Prob-